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NATURAL CONVECTION HEAT TRANSFER FROM ISOTHERMAL HORIZONTAL PLATES OF DIFFERENT SHAPES

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Abstract—Experiments were carried out on natural convection heat transfer from isothermal plates facing upwards in air in the range of $Gr \cdot Pr$ from 2×10^5 to 10^9. Both the average and local heat transfer were determined. Plates of different shapes (square, rectangular and circular) were used and "corner" and "edge" effects were investigated. Dimensionless equations are suggested for the laminar and turbulent regions.

NOMENCLATURE

- d, diameter of plate or square plate side-length;
- L, length of rectangular plate;
- h_c, corner average coefficient of heat transfer;
- h_e, edge average coefficient of heat transfer;
- h_t, turbulent average coefficient of heat transfer;
- h, average coefficient of heat transfer;
- $h_{l.c}$, local coefficient of heat transfer;
- h_{∞}, average coefficient of heat transfer of plate of infinite size;
- Nu, Nusselt number;
- Gr, Grashof number;
- Pr, Prandtl number.

INTRODUCTION

THE AVAILABLE data on heat transfer by natural convection from horizontal plates is limited. The equations suggested by Fishenden and Saunders [1] for square plates are used by almost all observers. The particulars of the work from which these equations were derived are, however, not given. Moreover no data seem to exist for other plate shapes. It is felt that a systematic investigation of the problem is needed. The present research work was carried out to fill a part of the existing gap by investigating the case of a horizontal plate facing upwards in air.

SUMMARY OF PREVIOUS WORK

Kraus [2], in 1940, found that the heat transfer from both the bottom and the top of horizontal square surfaces (from 16×16 cm to 26×26 cm) could be represented by the equation:

$$Nu = 0.137(Gr \cdot Pr)^{1/3}. \tag{1}$$

Fishenden and Saunders [1], in 1950, recommended the following equations for natural convection heat transfer from horizontal square surfaces facing upwards and downwards with a maximum size of about 2 ft and a temperature difference between surface and air up to 1000 deg. F. No reference is given for the work from which these equations were derived:

For plates facing upwards,

laminar region ($Gr \cdot Pr$ between 10^5 and 10^8)

$$Nu = 0.54(Gr \cdot Pr)^{1/4} \tag{2}$$

turbulent region ($Gr \cdot Pr$ higher than 10^8)

$$Nu = 0.14(Gr \cdot Pr)^{1/3}. \tag{3}$$

For plates facing downwards,

laminar region until $Gr \cdot Pr = 10^{10}$

$$Nu = 0.25(Gr \cdot Pr)^{1/4}. \tag{4}$$

Bosworth [3], in 1952, suggested the following equations for the heat transfer by natural convection from horizontal surfaces facing upwards without giving the background for his equations or the shape of the plate:

laminar region,

$$Nu = 0.71(Gr \cdot Pr)^{1/4} \tag{5}$$

turbulent region,

$$Nu = 0.17(Gr \cdot Pr)^{1/3}. \tag{6}$$

Mikheyev [4], in 1968, suggested, without giving any particulars as to the reference used, that the heat transfer by natural convection from a horizontal plate facing upwards was 30% more than that from a vertical plate of the same size in the same conditions. The heat transfer from a horizontal plate facing downwards was, however, 30% less than that from the vertical plate.

Husar and Sparrow [5], in 1968, photographically studied the shape of the natural convection flow adjacent to horizontal upward facing, electrically heated surfaces of different shapes (square, rectangular, triangular and circular). A flow visualization technique was employed which enabled the fluid motion to be visible by local changes of the pH value of the fluid. This permitted a study of the influence of the shape of the surface on the convection flow. For those shapes with corners a basic characteristic of the convection currents was found to be the partitioning of the flow field. The flow moved along parallel paths perpendicular to the edge of the plate.

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H. Kamal and A. Salah [6], in 1970, experimented with a rectangular plate 504 × 200 mm which was heated by hot or cold water at the back. The apparatus was designed to provide a constant temperature plate surface and to permit the measurement of the local heat transfer lengthwise. The plate was tested vertically, horizontally and at different angles of inclination with plate surface temperatures from 36 to 66°C, and Grashof numbers from 1.11×10^8 to 3.67×10^8.

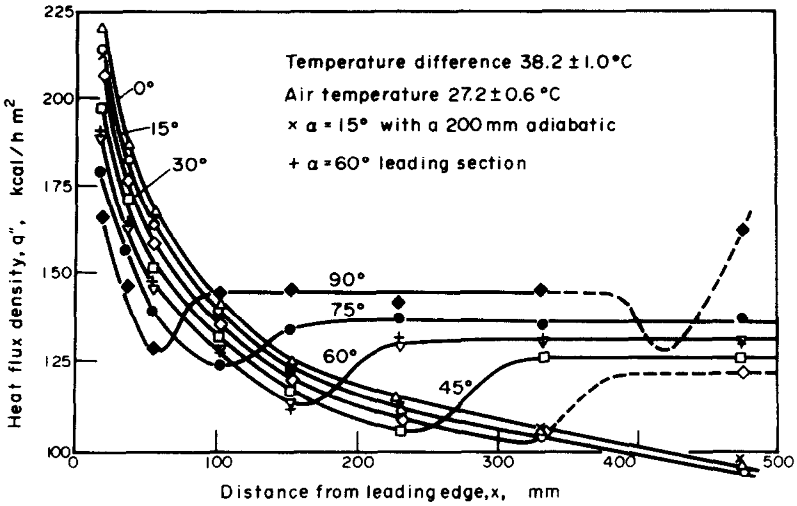

FIG. 1. Natural convection local heat flux density distribution for surface at different positive inclinations in air. (Hot surface, $\theta = 38.2$ deg. C [6].)

From the local heat flux vs plate length curves (Fig. 1) it was concluded that, for a plate of infinite size (for which case the edge effects could be neglected), the average heat transfer could be represented by:
For a plate facing upwards,

$$Nu = 0.135(Gr \cdot Pr)^{1/3} \tag{7}$$

For a plate facing downwards,

$$Nu = 0.068(Gr \cdot Pr)^{1/4}. \tag{8}$$

Luciano Pera and Benjamin Gebhart [7], in 1973, studied the stability of natural convection boundary-layer flow adjacent to horizontal and to slightly inclined surfaces. Some interferometry photographs were taken which confirmed the presence of a laminar boundary layer over the edges of horizontal plates facing upwards.

PATTERN OF FLOW OF CONVECTION CURRENTS

Figure 2 shows the idealized pattern of flow of the convection currents from a hot horizontal plate facing upwards in air as given by Mikheyev [4]. Figure 2(b) is for a plate with the edges in the laminar region. Beyond the edges the boundary layer separates and flows upwards. The area bounded by the edges gets its supply of air from upwards and is covered by turbulence.

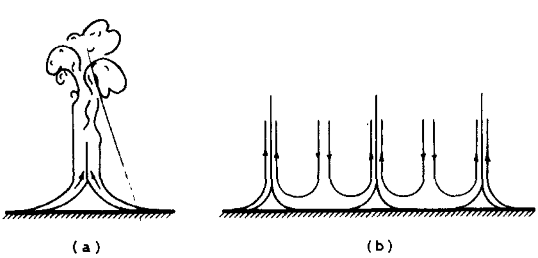

FIG. 2.

Figure 3 shows the idealized shape of convection current flow adjacent to the plate, for square, circular and rectangular plates, as deduced from the photographs taken by Husar and Sparrow [5]. The flow being normal to the edges, a square or rectangular plate of sufficient side length can be subdivided, as shown in Fig. 4, to corners, edges and a central region. This shows that there are three different average coefficients of heat transfer, h_c through the corners, h_e through the edges and h_t through the central region

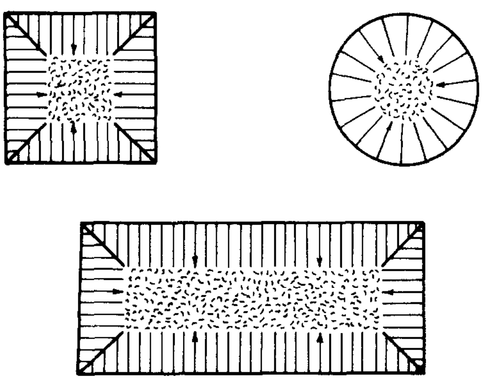

FIG. 3. Idealized pattern of flow of convection currents (arrows indicate direction of air flow).

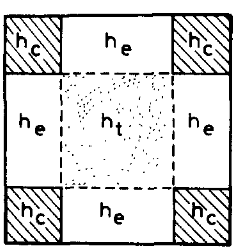

FIG. 4.

beyond the edge and corner effects. Needless to say that in the case of a circular plate of sufficient diameter, there will be no h_c and only h_e and h_t will exist. The apparatus used was designed to study these different values of coefficient of heat transfer.

APPARATUS

Figure 5 shows the principle of the apparatus used. It consists essentially of a plate (1) heated by steam at atmospheric pressure on the lower side. The plate forms the top of a compartment (2) which receives the steam through tube (3). A steam filled outer jacket (4)

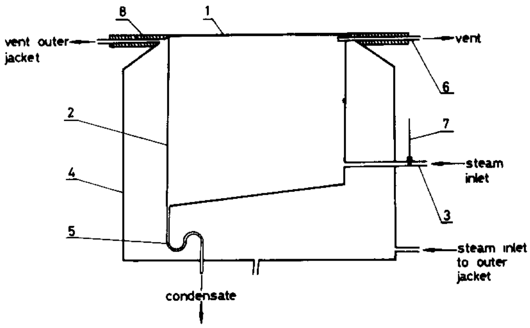

FIG. 5. Diagrammatic sketch of apparatus.

thermally insulates the compartment leaving only the top (the test plate) facing the atmosphere. The condensate in the compartment will then be due to the heat lost from the test plate alone. It could be collected from a syphon (5) fitted to trap the steam. A small vent tube (6) ensured atmospheric pressure in the compartment and the jacket. To make sure that the steam entered the compartment perfectly dry it was slightly superheated in an electric superheater. The amount of superheat was adjusted to about 8°C as shown by a mercury thermometer (7). This small amount of superheat could be neglected as it represented less than 1% of the heat given to the plate.

Three sets of apparatus were made. The first set consisted of eight square plates with side lengths equal to 50, 75, 100, 120, 150, 250, 300 and 450 mm respectively. The second set consisted of six circular plates with diameters equal to 100, 150, 250, 300, 400 and 500 mm respectively. The third set consisted of five rectangular plates all of which had a 150 mm width (smaller side) and lengths equal to 250, 300, 350, 450 and 600 mm respectively. The square and rectangular plates were made of 0.3 mm thick tin-sheet with the air-side surface polished bright to minimize radiation whereas the circular plates were made of 0.8 mm thick brass-sheet with the air-side surface nickel-plated and polished for the same reason. The emissivity of polished tin-sheet and of nickel-plated brass were taken from Mikheyev [4] as 0.05 and 0.08 respectively. The heat transfer due to radiation did not exceed 4% of the total heat transfer.

Two more plates, a square one with a side length equal to 450 mm, and a circular one with a diameter equal to 500 mm, were made in the form of concentric jackets as shown in Fig. 6 to enable the condensate, and therefore the heat transfer, from each jacket to be

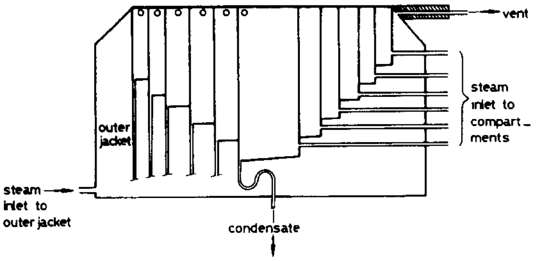

FIG. 6. Diagrammatic sketch of compartmental apparatus.

determined separately. Thus the variation of the local heat transfer across the plate could be determined.

The tested plates were inserted in wooden frames (8) fitted flush with plates. The experiments were carried out in a large room and the apparatus was protected from air currents by wooden shields.

The heat transfer on the steam-side surface of the test plate being much higher than that on the air-side surface, the temperature of the steam-side surface could be taken equal to the saturation temperature of the steam as registered by the barometer at the time of the experiment. The air-side plate surface temperature could then be found by calculating the drop of temperature in the plate metal by conduction. In fact this was so small that it could be neglected.

RESULTS AND DISCUSSION

Average heat transfer

1. *Square plates.* The condensate collected from the inner jacket of an apparatus of the form shown in Fig. 5 enabled average the heat transfer from the test plate to be calculated. Corrected for radiation, the coefficient of heat transfer by convection could be determined.

Figure 7 shows the results obtained for the square plates plotted in the form of log Nu against log $(Gr \cdot Pr)$ They can be represented by two straight lines, one with a slope equal to 1/4 (indicating the laminar region) followed by another with a slope equal to 1/3 (indicating the turbulent region). The laminar region extends to a value of $Gr \cdot Pr$ of about 4×10^7.

The laminar heat transfer ($Gr \cdot Pr$ from 2×10^5 to 4×10^7) can be represented, within $\pm 14\%$, by the equation:*

$$Nu = 0.70(Gr \cdot Pr)^{1/4}. \quad (9)$$

The turbulent heat transfer ($Gr \cdot Pr$ higher than 4×10^7) can be represented, within $\pm 12\%$, by the equation:*

$$Nu = 0.155(Gr \cdot Pr)^{1/3}. \quad (10)$$

Equation (9) gives heat-transfer values some 30% higher than the values given by equation (2) suggested by Fishenden and Saunders for the laminar region. However, it gives almost the same values as these given by equation (5) suggested by Bosworth. It is of interest to note that, according to the equations given by Fishenden and Saunders [1], the laminar heat transfer from a horizontal square plate facing upwards is some

*Physical properties are taken at mean film temperature.

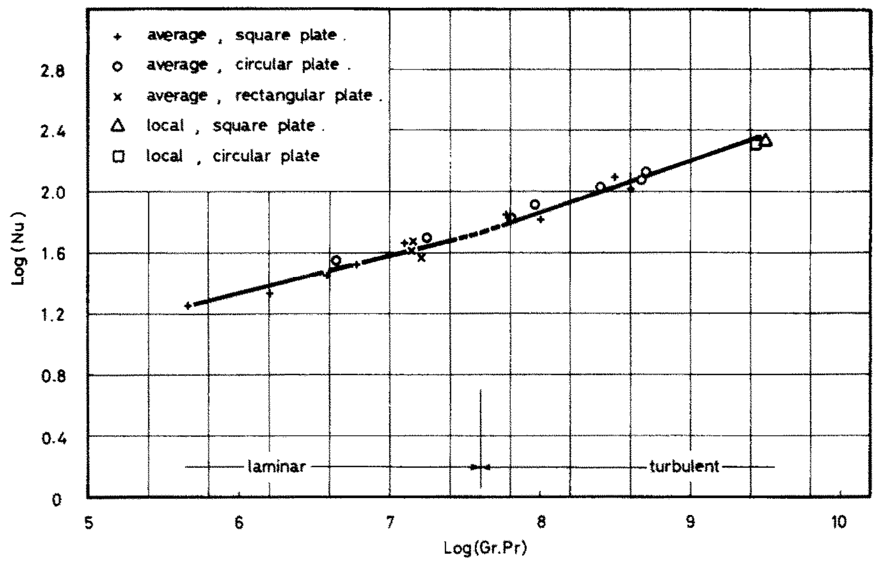

FIG. 7. Correlation of results of different plates.

4% less than the heat transfer from a vertical plate of the same size. This contradicts the findings of Mikheyev (4) which are in good agreement with the present results.

Equation (10) gives heat-transfer values 11% higher than the values given by equation (3) suggested by Fishenden and Saunders for the turbulent region but 11% lower than the values given by equation (6) suggested by Bosworth. It is worthy of note that the equations given by Fishenden and Saunders suggest turbulent heat-transfer values from horizontal plates facing upwards some 17% higher than the heat transfer from a vertical plate of the same size. Mikheyev [4], however, suggests that the horizontal plate would give 30% higher values. The present results lie in between the results of these observers.

2. *Circular plates.* The results obtained from the circular plates are also plotted in Fig. 7. It can be seen that they are well represented, within a maximum deviation of 14%, by equations (9) and (10) deduced from the square plate results. This has an important implication. As mentioned before the average heat transfer from a square plate depends on the corner effect, the edge effect and the plate side length. In the laminar region (Fig. 4) it is h_c, h_e and the side length which will fix the value of the total average heat transfer. As this total average has been found practically the same for both the square plate (which contains corner and edge effects) and the circular plate (which contains an edge effect only) the corner effect must be negligible.

3. *Rectangular plates.* The fact that corners were found to have a negligible effect on the total average heat transfer could also be checked by the experiments carried out on the rectangular plates.

All the plates had the same width (150 mm). This width was chosen to insure that the whole plate was in the laminar region. The lengths, however, were different as specified earlier.

The average heat-transfer coefficients are plotted in Fig. 8 against plate length. It will be seen that all the plates can be considered to give the same average coefficient of heat transfer (within ±12%). This confirms that corners have a negligible effect on the average heat transfer from rectangular plates and, therefore, this coefficient can be calculated from equation (9). The width (the shorter side length), in this case, is to be used as the characteristic dimension in calculating

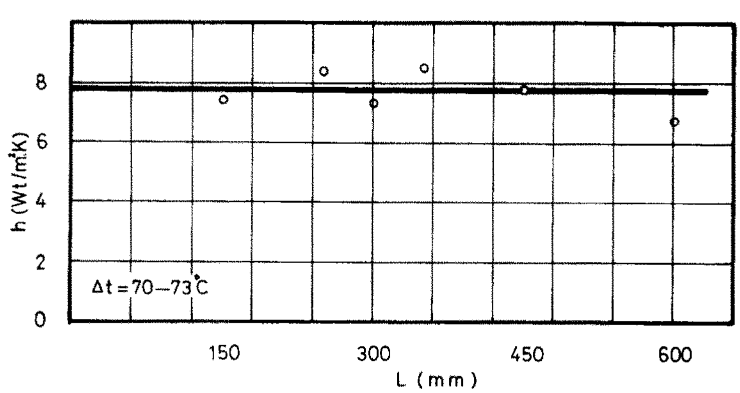

FIG. 8. Variation of rectangular plate coefficient of heat transfer with plate length.

Nu and *Gr*. The experimental results, thus calculated, are plotted in Fig. 7. It can be seen that they are in good agreement with the results from the other plates.

Needless to say, in the turbulent region equation (10) can also be used for rectangular plates.

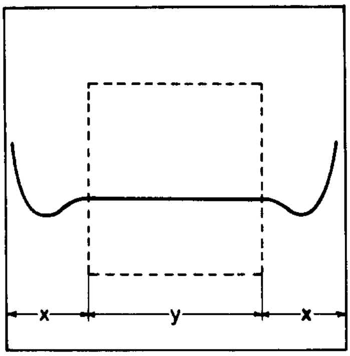

FIG. 9(a). General shape of variation of local heat transfer across plate.

deduced from the central part of the curve in Fig. 9, is:

$$Nu_{\infty} = 0.145(Gr \cdot Pr)^{1/3}. \quad (11)$$

Equation (11) gives heat-transfer values about 6% lower than those given by equation (10) obtained from the average heat-transfer experiments for the turbulent region.

Compared with equation (7) of Kamal and Salah the values given by equation (11) are some 7% higher. Despite the "apparent" agreement between the two equations it is worth mentioning that Kamal's plate was 504 × 200 mm. Judged by the *h* vs length curve shown in Fig. 1 the plate, widthwise, was all in the laminar region. The central part of the curve [from which equation (7) was deduced] can not, therefore, represent the turbulent heat transfer. This is a case of a long plate the width of which is not sufficient for the onset of turbulence.

The results obtained from the circular compartmental apparatus had the same characteristics as those

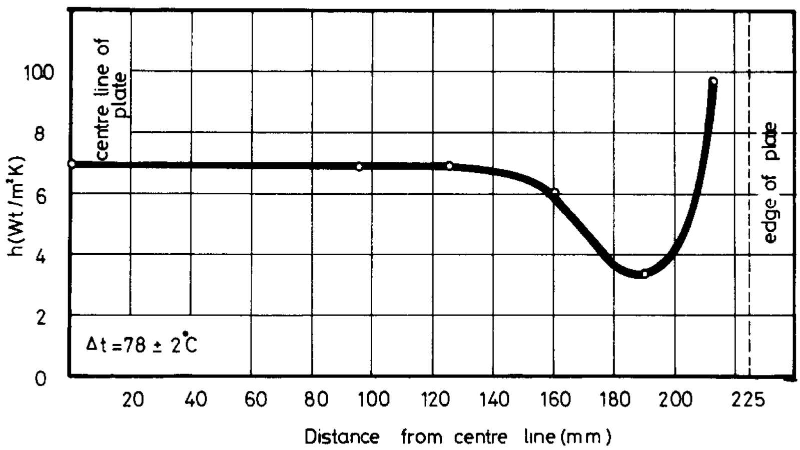

FIG. 9(b). Local heat-transfer coefficient, square plates.

Local heat transfer

To complete the study of the problem two multi-compartmental test chambers were made as shown in Fig. 6. One of them was square and the other was circular. The condensate could be collected separately from the concentric compartments. This enabled the local heat transfer for each section of the plate to be calculated.

Figure 9 shows the results obtained from the square apparatus. The curve shows the variation of the local heat transfer $h_{l.c}$ across the plate. The variation of $h_{l.c}$ along that part of the curve marked *X* is due to the combined edge and corner effect. It represents the laminar and the transition regions in which the value of $h_{l.c}$ varies from point to point. The part marked *Y*, however, represents a region beyond these effects. $h_{l.c}$ in this part is independent of the distance from the plate edge indicating the turbulent condition.

The laminar region extends from the plate edge until a value of $(Gr \cdot Pr)_{l.c}$ equal to about 2×10^5 is reached. A transition region follows. Beyond a value of $(Gr \cdot Pr)_{l.c}$ equal to about 3.8×10^6 turbulence exists.

The equation representing the turbulent region, as obtained from the square compartmental one. They could also be represented by equation (11). This is indeed as it should be because, in the turbulent region, the shape of the plate can have no effect on the heat transfer if the plate is infinitely large.

CONCLUSIONS

Average heat transfer

1. The experiments covered the laminar and the turbulent regions. The laminar region extends up to a value of $(Gr \cdot Pr)$ equal to 4×10^7.

2. The equation obtained for the laminar region gives heat-transfer values 30% higher than the values given by the equation suggested by Fishenden and Saunders but in agreement with the values given by Bosworth's equation and with Mikheyev's recommendation.

3. The equation obtained for the turbulent region gives heat transfer values 11% higher than the values given by Fishenden and Saunders' equation, but 11% lower than those given by Bosworth's equation.

4. Corners have a negligible effect on the average heat transfer from the plate.

5. The average heat-transfer coefficient from a circular plate can be considered equal to the average heat-transfer coefficient from a square plate with side length equal to the diameter of the circular plate.

6. The average heat-transfer coefficient from a rectangular plate can be considered equal to the average heat-transfer coefficient from a square plate with side length equal to the widths of the rectangular plate.

Local heat transfer

7. The local heat-transfer experiments showed a laminar region which extends from the plate edge until a value of $(Gr \cdot Pr)_{l.c}$ equal to about 2×10^5. A transition region follows. Beyond a value of $(Gr \cdot Pr)_{l.c}$ equal to about 3.8×10^6 turbulence exists.

8. The equation obtained from the local heat-transfer experiments for the turbulent region confirmed that obtained from the average heat transfer experiments.

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TRANSFERT DE CHALEUR PAR CONVECTION NATURELLE SUR DES PLAQUES HORIZONTALES DE FORMES DIFFERENTES

Résumé—Des expériences de transfert de chaleur par convection naturelle ont été effectuées dans l'air sur des plaques isothermes tournées vers le haut, pour un domaine de variation de $Gr \cdot Pr$ allant de 2.10^5 à 10^9. Le transfert de chaleur local et le transfert moyen sont tous deux déterminés. Des plaques de formes différentes (carrées, rectangulaires et circulaires) ont été utilisées et les effets "d'angle" et de "bord" ont été analysés. On propose des équations adimensionnelles décrivant les régions laminaire et turbulente.

DER WÄRMEÜBERGANG BEI NATÜRLICHER KONVEKTION AN ISOTHERMEN, HORIZONTALEN PLATTEN VERSCHIEDENARTIGER FORM

Zusammenfassung—Es wurden Versuche über den Wärmeübergang bei freier Konvektion an der Oberseite isothermer Platten mit Luft im Bereich von $2 \times 10^5 < Gr \cdot Pr < 10^9$ durchgeführt. Sowohl der mittlere wie der lokale Wärmeübergang wurde bestimmt. Es wurden Platten verschiedener Form (quadratisch, rechteckig, kreisförmig) verwendet und der Ecken- und Seiteneinfluß untersucht. Sowohl für den laminaren wie für den turbulenten Bereich werden dimensionslose Gleichungen vorgeschlagen.

ПЕРЕНОС ТЕПЛА ЕСТЕСТВЕННОЙ КОНВЕКЦИЕЙ ОТ ИЗОТЕРМИЧЕСКИХ ГОРИЗОНТАЛЬНЫХ ПЛАСТИН РАЗЛИЧНОЙ ФОРМЫ

Аннотация — Проведены эксперименты по переносу тепла при естественной конвекции от изотермических пластин, с одной стороны омываемых воздухом, в диапазоне чисел $Gr \cdot Pr$ от 2×10^5–10^9. Определялись как средние, так и локальные коэффициенты переноса тепла. Использовались пластины различной формы (квадратные, прямоугольные и круглые) и исследовались «угловые» и «краевые» эффекты. Предложены безразмерные уравнения для ламинарной и турбулентной областей.